American Journal of Engineering Science and Technology Research
Vol. 1, No. 5, June 2013, PP: 88-95 , ISSN: 2327-8269 (Online)
Available online at www.ajestr.com

Research article

MATHEMATICAL MODEL TO PREDICT THE RATE OF PERMEABILITY AND POROSITY AT DIFFERENT TEMPERATURES OF CEMENT IN CONCRETE STRUCTURES

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Abstract

Mathematical model to predict the rate of porosity and permeability in concrete structure has been evaluated The model were developed to determine the rate permeability and porosity deposition in concrete structure, permeability remain the ease of fluid passing through concrete, through the microstructural pore space in concrete composition, while porosity remain the volumetric proportion of void, such extensive condition are from the properties of concrete. The rates of these two parameters in concrete structure determine the durability and performance of concrete. The model were derived to establish a model prediction for porosity and permeability in concrete structures the behaviour of concrete characteristics were considered in the system, the binding agent cement paste were thoroughly expressed in the system, because it definitely determine the bond of the concrete characteristics thus structural performance and durability. The developed model will definitely predict the rate of permeability and porosity of concrete structure; this will definitely reflect the performance and durability of concrete structures.

Keywords: mathematical model, permeability, porosity and temperatures in concrete structures.

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1. Introduction

Proportion of Mix design for pervious concrete in chilly weather climates was developed and reported (NRMCA, 2004). The desired void content can be achieved by modifying the level of compacting effort or by adjusting the aggregate gradation, proportions and properties. Single-sized aggregate could provide concrete with high porosity but lower strength. Inclusion of sand and latex can improve the workability of Portland cement pervious concrete

(Kevern, et al, 2005). The effect of water quality on properties of pervious concrete was reported (Park and Tia, 2004).

The pore structure of pervious concrete was evaluated by image analysis (Neithalath and Weiss, 2003). Above a certain level, increasing cement content results in a reduced concrete porosity with insignificant influence on concrete strength. The addition of sand and latex significantly improved the workability and resulted in higher strength, appropriate permeability, and freeze-thaw resistance. Chemical admixtures can improve the workability significantly; obtain the higher strength at relative lower cement contents and results in relative higher porosity (Huang, 2006 Kevern, et al, 2005, Yang and Jiang, 2003).

Permeability of concrete is not so much a function of the porosity as it is a function of the size, distribution and the continuity of capillary pores. High quality concrete at an early age has a total void volume of about 20 to 25% and about 10 to 25% at a mature age (OberholsterOberholster1986. Haynes, 1980, Mehta and Manmohan Mehta and Mamohan, 1980) observed that the permeability of hardened cement paste is negligible (irrespective of its porosity) unless a network of micropores wider than a specific diameter of about 100 nm exists in the system. The volume of pores in most natural aggregates is usually under 3% and rarely exceeds 10% (Mehta, 1993), while the capillary pores in a typical cement paste range from 20 to 40%. Many researchers have attempted to develop the applicable relationship between permeability and pore pressure since (Powers et al. 1958) first reported the relations between permeability and porosity Nayme and Illston (Nayme,1980) reported an influence of maximum continuous pore radius on permeability. Hughes (Hughes, 1985) reported the influence of average pore radius on permeability by using the Poiseuille formula. (Mehta and Monteiro, 1993) took pore size distribution into account and proposed an empirical formula to express the relationship between permeability and pore structure. A model relating permeability of cement paste with its pore structure, determined by MIP, has been reported by Cui and Cahyadi (Dullien, 1979). Kumar and Bhattacharjee (Kumar and Bhattacharjee 2004) have drawn the following conclusions from their research work on assessment of permeation quality of concrete through MIP [Kumar and Bhattacharjee, 2004]: (i) the permeation quality of concrete can be assessed on the basis of the knowledge of porosity and pore system characteristics of concrete such as equivalent pore radius and mean distribution pore radius of the concrete obtained through MIP results; (ii) the relationship involving mean distribution pore radius of the pores yields a better correlation than that involving equivalent pore radius; (iii) MIP can be used as an additional method for assessing the permeation quality of concrete. As a fluid flows essentially through the interconnected pore path in a porous material such as concrete, the flow path is tortuous in nature. It is assumed that all effects arising from pore orientation, connectivity, size variation, etc. can be lumped together as a single *"tortuosity factor"*. This is obviously a gross assumption since it depends on the implicit assumption that the effects of pore geometry and structure are the same for all pore sizes. Since the tortuosity factor is a structural parameter, it should not depend on the sorbate or the experimental conditions (Karger and Ruthwen 1986) Dullien (Dullien, 1979) has also shown that if the pore structure is characterized in sufficient detail, a reasonably accurate prediction of the tortuosity factor can be made. However, this requires a detailed knowledge of the pore shape as well as pore size distribution. In practice, it is generally simpler to treat the tortuosity as an empirical constant [13].

2. Theoretical background

The rate concrete permeability are determine on the concrete characteristics, concrete is known to be composite material formed by mixing and curing ingredients such as s cement fine , coarse aggregate and water, More so concrete contains other additional ingredients such as chemical admixtures, this are air entraining admixture, fly ash fibers, slag and other products. concrete deposits physical and chemical including durability characteristics of concrete, the depend on several factors, this are determined by the amount of the components, temperature, pore size distribution, surface area, interfacial features, exposure conditions consequently, a better understanding in cementitious system are imperative on the application of diverse techniques. Concrete comprises large quantity of manmade materials, but the active constituents of concrete are the cement past. The performance of concrete depends on the quality of the materials in the concrete composition, this include their proportion placement, and exposed conditions, but the focus of this study is on permeability and porosity of the concrete, when concrete are placed either as insitu or precast conditions, curing of the concrete are done for the concrete to attained it optimum strength, the material use for curing of this concrete is quality water that is free of any contaminants, this is to avoid the degradation of the concrete strength. The continuity of water from different sources will definitely continue to reduce the strength on the concrete; this is through the percentage of permeability and porosity that the concrete structures deposit the rate of porosity and permeability depend on the rate of compaction these are reflected from the mix proportion on the concrete structures because the mix proportion determine the strength of the concrete, to monitor the rate permeability and porosity in concrete structure, mathematical model were developed, the model are from formulated mathematical equation through the parameters that determined the rate of permeability and porosity of concrete structures, the equation were derived to produce a model that will monitor the rate of permeability and porosity in concrete structures, the governing equation derived are expressed bellow.

3. Governing equation

$$P\frac{\partial c}{\partial t} = \phi\alpha\frac{\partial c}{\partial z} + Tc\frac{\partial c}{\partial z} \qquad \text{.............................} \quad (1)$$

Equation (1) is the governing equation to predict the rate of permeability and porosity in concrete structure at different temperature, the equation were development in accordance with the influential parameters that influence the deposition of porosity and permeability in the concrete structure. Temperatures were found to influence the strength of concrete at different conditions; these conditions were considered as it is expressed in the governing equations.

Nomenclature

P	=	Coefficient of permeability in (m/s)
C	=	Particle concentration [ML^{-3}]
$\phi\alpha$	=	porosity and Viscosity of the fluid [poise] as a function of [ML^{-3}]

Tc = Temperature [^{0}c]

X = Distance [L]

T = Time of concentration [T]

Using $C = ZT$ as solution for equation (1)

$$PZT^1 = \phi\alpha Z^1 T + TcZ^1 T \quad \ldots\ldots\ldots\ldots \quad (2)$$

Dividing equation (2) by ZT

$$P\frac{T^1}{T} = \phi\alpha\frac{Z^1}{Z} + Tc\frac{Z^1}{Z} \quad \ldots\ldots\ldots\ldots \quad (3)$$

From equation (3) we have

$$P\frac{T^1}{T} = -\lambda^2\phi \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$PT^1 + \lambda^2 T = 0 \quad \ldots\ldots\ldots\ldots \quad (5)$$

Also from equation (3) we have

$$PT^1 = \phi\alpha\frac{Z^1}{Z} + Tc\frac{Z^1}{Z} \quad \ldots\ldots\ldots\ldots \quad (6)$$

Also from (2) we have

$$\phi\alpha\frac{Z^1}{Z} + Tc\frac{Z^1}{Z} = \lambda^2 \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$\phi\alpha\frac{Z^1}{Z} + Tc\frac{Z^1}{Z} = 0 \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$Z^1 + Z^1 + \frac{1}{\phi\alpha}Z = 0 \quad \ldots\ldots\ldots\ldots \quad (9)$$

$$Z^1 + Z^1 + \beta Z = 0 \quad \ldots\ldots\ldots\ldots \quad (10)$$

Where $\beta = \dfrac{1}{\phi\alpha}$ ………………………. (11)

Separation of variables were applied in the system, this is to ensure that parameters that influence the system express their function, thus subject them self to other parameters were there relationship between each other will be thoroughly expressed, this concept were applied, as express from equation (1) to (11).

Suppose $X = \ell^{Mz}$ in (10)

$$X^1 M_1 \ell^{Mz},\ X^1 = M^2 \ell^{Mz} \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$XM^2\ell^{Mz} + M\ell^{Mz} - \beta\ell^{Mz} = 0 \quad \ldots\ldots\ldots\ldots \quad (13)$$

$$\left(XM^2 + M - \beta\right)\ell^{Mz} = 0 \quad \ldots\ldots\ldots\ldots \quad (14)$$

But $\ell^{M_z} \neq 0$

$$XM^2 + M - \beta = 0 \quad \ldots\ldots\ldots\ldots \quad (15)$$

Quadratic expression were applied at this level of derivation of the governing equation, the essence of this method were to descretize there functions at different condition, the behaviour of the concrete mix are determined by the reaction of characteristics of the microstructural element of the concrete structures, this include there setting time and their reaction through the hydration from the cement past. The expression from quadratic concept will streamline the variation of the behaviour of the structure elements, this condition reflect on the setting time and the level of strength it attained.

Applying quadratic expression, we have

$$M_{1,2} = \frac{-1 \pm \sqrt{1+4\beta z}}{2z} \quad \ldots\ldots\ldots\ldots \quad (16)$$

$$M_1 = \frac{-1 + \sqrt{1+4\beta z}}{2z} \quad \ldots\ldots\ldots\ldots \quad (17)$$

$$M_2 = \frac{-1 - \sqrt{1+4\beta z}}{2z} \quad \ldots\ldots\ldots\ldots \quad (18)$$

Therefore, $X_{(Z)} = C_1 \ell^{M_z} + C_2 \beta \ell^{M_{2z}}$ $\quad \ldots\ldots\ldots\ldots \quad (19)$

$$= C_1 Cos M_{1Z} + C_2 Sin M_{2Z} \quad \ldots\ldots\ldots\ldots \quad (20)$$

Solving from equation (5)

$$\boxed{T_{(t)} = T_{(o)} \ell^{\frac{-\lambda^2}{P}t}} \quad \ldots\ldots\ldots\ldots \quad (21)$$

The expressed model at this condition are reflected from the concrete properties, this is through the structural characteristics of the concrete, the model in (21) are expresses to reflect the setting time of concrete , the concrete continue to attain strength from the binding agent thus cement past. The expression reflect the time when the concrete are composed and place thus it begin to attained its optimum strength, base on the constituents of the materials in the concrete structures. The expressed model will be able to monitor the concrete attaining strength under the influence of time.

Hence the solution of equation (21) becomes

$$\boxed{C_{(Zt,t)} = \left(C_1 \, Cos M_{1Z} + C_2 \, Sin \, M_{2Z}\right) \ell^{\frac{-\lambda^2}{P}t}} \quad \ldots\ldots\ldots\ldots \quad (22)$$

The expressed model equation in (22) were subjected to thickness of the concrete structure were length is established including the transportation of the concrete to where it will be placed, the process of transporting the concrete, the workability of the concrete may begin to deteriorate, even though it is thoroughly mixed, that is why its proper to establish thorough compaction so that the concrete maintained its fresh mix to where it will be place the model equation in (22) express the concrete structure under the influence of time and distance in the system.

But if $x = \frac{v}{t}$

Therefore, equation (22) can be written as

$$\boxed{C_{(Zt,t)} = \left(C_1 \, Cos M_1 \frac{v}{t} + C_2 \, Sin \, M_2 \frac{v}{t} \right) \ell^{\frac{-\lambda^2}{P}t}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Equation (23) is the final model that express the prediction of permeability and porosity percentage in concrete structure, the model considered several conditions in concrete setting, the microstructural element from the concrete characteristics were considered in the system, this condition is to ensure that the rate of permeability in structural behaviour are express as it reflect on the compressive strength of the concrete , the system were able to consider the velocity at short time and distance with respect to the cement paste from the fluid binding the composition of concrete material together. The conditions are reflected from velocity of the fluid in fresh concrete mix as expressed in (23). The model developed will determine the rate permeability and percentage of porosity in any structural component the model predict the rate of the two parameters at different temperature in concrete mix.

4. Conclusion

Mathematical model to predict the rate of permeability and porosity has been evaluated the model were developed through the expressed governing equation, several conditions that influences the system, thus the rate of permeability and porosity in concrete structure were expressed. The models will provide a platform to known the variation of porosity and percentage in concrete structures, variations of concrete are determined from the cement paste, the variation from cement composition determined the major strength of cement, this is through the binding agent, `performance of concrete depend on the quality of the ingredients, their proportion, placement and exposure of cement conditions. Quality of raw materials used for the manufacture clinker, the calcining conditions, the fines and particle size of the cement, the relative proportions of the phase including the hardened cement paste, and the amount of the mixing water, influence the physiochemical behaviour of the hardened cement paste. In the fabrication of concrete, amount to different type of cement, fine and coarse aggregate, water, temperature of mixing, admixture, chemical, and durability behaviour. Concrete structures under goes this condition when they maintained the standard principle, the condition reflect the setting time of the binding agent the setting time of this material are through the paste or mortar fraction , these are through the initial and fine setting time of cement paste. Such condition are expressed in concrete durability through the performance of the material under the influence of the mix proportion and the placement, these condition determine the performance of concrete structure, the established model to monitor permeability and porosity of concrete are reflected from the concrete characteristics condition and its compositions. The expressed developed model can be applied to monitor the percentage of permeability and porosity in concrete structures.

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