

Research article

PREDICTIVE MODEL TO MONITOR THE RATE OF BULK DENSITY IN FINE AND COARSE SOIL FORMATION INFLUENCED VARIATION OF POROSITY IN COASTAL AREA OF PORT HARCOURT

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Abstract

Predictive model to monitor the rate of bulk density in fine and coarse soil formation influenced variation of porosity has been articulated. Soils are often identified and classified by their arrangement and element size. Though, bulk density and pore space or porosity provides a more functional corporeal explanation of soil. Soil bulk density is a measure of soil compaction and strength. Porosity also provides some approximation of compaction and the utmost space obtainable for water (at saturation) or air. Air-filled porosity (**fa**) is useful for many soil-related investigations and has been found to be a good indicator of soil biological and chemical activities. This can be obtained by the difference between the porosity (**f**) and the volumetric water content. Such expressions are established relationship between the stated parameters, the established model developed will definitely predict the rate of fine and coarse sand under the influence of porosity in coastal area of Port Harcourt. **Copyright © AJESTR, all rights reserved.**

Keywords: predictive model, bulk density, fine and coarse soil, and variation of porosity

1. Introduction

Bulk density is a measure of the weight of the soil per unit volume (g/cc), usually given on an oven-dry (110° C) basis. Variation in bulk density is attributable to the relative proportion and specific gravity of solid organic and inorganic particles and to the porosity of the soil. Most mineral soils have bulk densities between 1.0 and 2.0. Although bulk densities are seldom measured, they are important in quantitative soil studies, and measurement should be encouraged. Such data are necessary, for example, in calculating soil moisture movement within a profile and rates of clay formation and carbonate accumulation. Even when two soils are compared qualitatively on the basis of their development for purposes of stratigraphic correlation, more accurate comparisons can be made on the basis of total weight of clay formed from 100 g of parent material than one percent of clay alone. To convert percent to weight per unit volume, multiply by bulk density (Birkeland, 1984). The determination usually consists of drying and weighing a soil sample, the volume of which is known (core method) or must be determined (clod method and excavation method). These methods differ in the way the soil sample is obtained and its volume determined

A different principle is employed with the radiation method. Transmitted or scattered gamma radiation is measured; and with suitable calibration, the density of the combined gaseous-liquid-solid components of a soil mass is determined. Correction is then necessary to remove the components of density attributable to liquid and gas that are present. The radiation method is an in situ method (Blake and Hartge, 1986). Clod and core methods have been used for many years. Excavation methods were developed in recent years, chiefly by soil engineers for bituminous and gravelly material. More recently the excavation method has found use in tillage research where surface soil is often too loose to allow core sampling, or where abundant stones preclude the use of core samplers. Radiation methods have been used since the 1950's, especially in soil engineering (Blake and Hartge, 1986). All the earlier mentioned methods have advantages and disadvantages according to the samples that are available and the sampling method. This method of discussion here is the clod method. The bulk density of clods, or peds, can be calculated from their mass and volume. The volume may be determined by coating a clod of known weight with a water-repellent substance and by weighing it first in air, then again while immersed in a liquid of known density, making use of Archimedes' principle. The clod or peds must be sufficiently stable to cohere during coating, weighing and handling (Blake and Hartge, 1986). The clod method is applied commonly by pedologists or paleopedologists.

Bulk density is dependent on soil texture and the densities of soil mineral (sand, silt, and clay) and organic matter particles, as well as their packing arrangement. As a rule of thumb, most rocks have a bulk density of 2.65 g/cm³ so ideally, a medium textured soil with about 50 percent pore space will have a bulk density of 1.33 g/cm³. Generally, loose, porous soils and those rich inorganic matters have lower bulk density. Sandy soils have relatively high bulk density since total pore space in sands is less than that of silt or clay soils. Fine-textured soils, such as silt and clay loams, that have good structure have higher pore space and lower bulk density compared to sandy soils. Bulk density typically increases with soil depth since subsurface layers have reduced organic matter, aggregation, and root penetration compared to surface layers and therefore, contain less pore space. Subsurface layers are also subject to the compacting weight of the soil above them. The wetting and drying and freeze/thaw cycles that occurs in soils naturally, generally do very little to alter soil bulk density. Bulk density is changed by crop and land management

practices that affect soil cover, organic matter, soil structure, and/or porosity. Plant and residue cover protects soil from the harmful effects of raindrops and soil erosion. Cultivation destroys soil organic matter and weakens the natural stability of soil aggregates making them susceptible to damage caused by water and wind. When eroded soil particles fill pore space, porosity is reduced and bulk density increases. Cultivation can result in compacted soil layers with increased bulk density, most notably a “plow pan” Livestock and agricultural and construction equipment exert pressure that compacts the soil and reduces porosity, especially on wet soils. Bulk density reflects the soil’s ability to function for structural support, water and solute movement, and soil aeration. Bulk densities above thresholds indicate impaired function. Bulk density is also used to convert between weight and volume of soil. It is used to express soil physical, chemical and biological measurements on a volumetric basis for soil quality assessment and comparisons between management systems. This increases the validity of comparisons by removing error associated with differences in soil density at time of sampling. High bulk density is an indicator of low soil porosity and soil compaction. It may cause restrictions to root growth, and poor movement of air and water through the soil. Compaction can result in shallow plant rooting and poor plant growth, influencing crop yield and reducing vegetative cover available to protect soil from erosion. By reducing water infiltration into the soil, compaction can lead to increased runoff and erosion from sloping land or waterlogged soils in flatter areas. In general, some soil compaction to restrict water movement through the soil profile is valuable under arid conditions, but under humid conditions compaction decreases yields. The bulk compactness of different soils varies based largely on soil texture and the degree of soil compaction. Sandy soils with low organic matter tend to have higher bulk density than clayey or loamy soils. Soil bulk density is usually higher in subsurface soils than in surface horizons, in part due to compaction by the weight of the surface soil. The bulk solidity indirectly provides a gauge of the soil porosity (amount of pore space). Soil porosity is the ratio of the volume of soil pores to the total soil volume. In general, clayey soils have an abundance of very small pores (microspores) that give them a higher *total porosity* compared to sands, which are dominated by larger, but *fewer* pores. Consider the relative sizes of a single sand grain and several clay particles existing as an aggregate. Low porosity tends to inhibit root penetration, water movement, and gas movement. Soils are compacted to improve the stability of fills – reducing the likelihood of failures and enhancing safety. Soil fills settle and compress over time. The amount of settlement depends upon the initial compaction rate, among other things. Foundations of heavy buildings, highway roadbeds, and airport runways all require considerable levels of soil compaction for satisfactory performance. Construction of earth-fill dams also involves heavy compaction to provide stable slope faces as well as a uniform and controlled rate of seepage through the dam core. The degree of necessary compaction is less clear in earth fills along stream banks because of conflicting project objectives and allowable factors of safety that differ from the examples above. The effects of soil compaction on the soil strength, compressibility, hydraulic conductivity, and structure have been well-studied (Assouline et al. 1997, Bowles 1992, Lambe and Whitman 1969, Seed and Chan 1959) and a series of standardized testing procedures have become widely adopted by professionals (Hunt 1986). One of the original and most popular tests, the standard compaction test, was developed by R. R. Proctor in the 1930’s. The procedure involves compacting three sequential layers of soil in a 4-in-diameter mold with a volume of $1/30 \text{ ft}^3$, using a 5-1/2-lb hammer dropped 25 times from a height of 12 in. The density that can be achieved using this fixed energy of compaction is dependent upon both the

textural composition of the soil and its moisture content at the time of the test. The density of the soil is achieved through the close packing of the particles. The lubrication effect of an optimal moisture level allows soil particles to become more easily realigned during the compaction procedure, leading to the highest degrees of compaction. For any given textural composition of soil, there is a maximum dry density that can be achieved at the optimal moisture level using the standard Proctor test. In general, compacted granular soils will have higher dry densities in the range of 115 to 135 lb/ft³ (1.84 to 2.16 g/cm³) than those of clayey to silty soils, which are in the range of 85 to 115 lb/ft³ (1.36 to 1.84 g/cm³). The corresponding optimum moisture contents for the granular and silty to clayey soils are generally on the order of 5 to 15 percent and 20 to 35 percent, respectively (Abramson et al. 1995). Once compacted to the selected degree, various parameters of soil strength, including saturated and suction cohesion as well as effective stress envelope, vary with soil type, but all are considerably improved over the uncompacted state (Hunt 1986). Sandy soils have large continuous pores, while clays have small pores, which transmit water slowly. Clays, however, contain more pore space than sandy soil for growing plants, pore sizes are more important than total pore space. Therefore, plants will have a better environment in sandy soils if porosity is low because of the increase in water retention. Coppin and Richards (1990) agree that the critical dry density depends on the soil texture and suggest values of about 87 lb/ft³ (1.4 g/cm³) for clay soils and 106 lb/ft³ (1.7 g/cm³) for sandy soils.

Theoretical Background

Theoretical background for 3rd degree polynomial curve fitting

$$\text{General: } y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

If the above polynomial fits the pair of data (x, y) it means that every pair of data will satisfy the equation (polynomial).

$$\text{Thus; } y_1 = a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 + \dots + a_nx_1^n \quad (1)$$

$$y_2 = a_0 + a_1x_2 + a_2x_2^2 + a_3x_2^3 + \dots + a_nx_2^n \quad (2)$$

$$y_3 = a_0 + a_1x_3 + a_2x_3^2 + a_3x_3^3 + \dots + a_nx_3^n \quad (3)$$

$$y_4 = a_0 + a_1x_4 + a_2x_4^2 + a_3x_4^3 + \dots + a_nx_4^n \quad (4)$$

Summing all the equations will yield

$$\sum_{i=1}^{i=n} y_i = \sum a_0 + \sum_{i=1}^{i=n} a_1 x_i + \sum_{i=1}^{i=n} a_2 x_i^2 + \sum_{i=1}^{i=n} a_3 x_i^3 + \sum_{i=1}^{i=n} a_4 x_i^4 + \dots + \sum_{i=1}^{i=n} a_n x_i^n$$

$$\sum_{i=1}^{i=n} y_i = na_0 + a_1 \sum_{i=1}^n x_i + a_2 \sum_{i=1}^n x_i^2 + a_3 \sum_{i=1}^n x_i^3 + \dots + \sum_{i=1}^n x_i^n \quad (5.)$$

To form the equations to solve for the constants $a_0, a_1, a_2, a_3, \dots, a_n$.

We multiply equations (5) by $x_i, x_i^2, x_i^3, \dots, x_i^n$.

Multiply equation (6) by x_i

$$\begin{aligned} x_i \sum y_i &= na_0 x_i + a_1 x_i \sum x_i + a_2 x_i \sum x_i^2 + a_3 x_i \sum x_i^3 + \dots + a_n x_i \sum x_i^n \\ \sum y_i x_i &= a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3 + a_3 \sum x_i^4 + \dots + a_n \sum x_i^{n+1} \end{aligned} \quad (7)$$

Multiply equation (6) by x_i^2

$$\begin{aligned} x_i^2 \sum y_i &= na_0 x_i^2 + a_1 x_i^2 \sum x_i + a_2 x_i^2 \sum x_i^2 + a_3 x_i^2 \sum x_i^3 + \dots + a_n x_i^2 \sum x_i^n \\ \sum y_i x_i^2 &= a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4 + a_3 \sum x_i^5 + \dots + a_n \sum x_i^{n+2} \end{aligned} \quad (8)$$

Multiply equation (3.84) by x_i^3

$$\begin{aligned} x_i^3 \sum y_i &= na_0 x_i^3 + a_1 x_i^3 \sum x_i + a_2 x_i^3 \sum x_i^2 + a_3 x_i^3 \sum x_i^3 + \dots + a_n x_i^3 \sum x_i^n \\ \sum y_i x_i^3 &= a_0 \sum x_i^3 + a_1 \sum x_i^4 + a_2 \sum x_i^5 + a_3 \sum x_i^6 + \dots + a_n \sum x_i^{n+3} \end{aligned} \quad (9)$$

Multiply equation (5,6 and 7) by x_i^n

$$\begin{aligned} x_i^n \sum y_i &= a_0 n x_i^n + a_1 x_i^n \sum x_i + a_2 x_i^n \sum x_i^2 + a_3 x_i^n \sum x_i^3 + \dots + a_n x_i^n \sum x_i^n \\ &= a_0 \sum x_i^n + a_1 \sum x_i^{n+1} + a_2 \sum x_i^{n+2} + a_3 \sum x_i^{n+3} + \dots + a_n \sum x_i^{n+n} \end{aligned} \quad \dots n$$

Putting equations (5, 6, 7, 8, and 9) to n into matrix form

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 & \dots & \sum x_i^n \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \dots & \sum x_i^{n+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \dots & \sum x_i^{n+2} \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 & \dots & \sum x_i^{n+3} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \sum x_i^n & \sum x_i^{n+1} & \sum x_i^{n+2} & \sum x_i^{n+3} & \dots & \sum x_i^{n+n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \dots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i^2 \\ \sum y_i x_i^3 \\ \dots \\ \sum y_i x_i^n \end{bmatrix}$$

Solving the matrix equation yields values for constants $a_0, a_1, a_2, a_3, \dots, a_n$ as the case may be depending on the power of the polynomial.

From the above matrix; for our particular case; i.e. polynomial of the third order:

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 \tag{11}$$

The equivalent matrix equation will be; ($n = 3$).

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i^2 \\ \sum y_i x_i^3 \end{bmatrix}$$

4. Results and Discussion

Results and tables of predictive and measured values of bulk densities are presented in tables and figures below

Table: 1 predicted and Measured values of bulk density at different depths

| Depth M | Predicted Values g/cm3 | Measured Values g/cm3 |
|---------|------------------------|-----------------------|
| 200 | 2.13 | 2.2 |
| 400 | 2.11 | 2.22 |
| 600 | 2.07 | 2.18 |
| 800 | 2.02 | 1.99 |
| 1200 | 1.88 | 1.89 |
| 1400 | 1.8 | 1.86 |
| 1600 | 1.71 | 1.69 |

| | | |
|------|-------|-------|
| 1800 | 1.61 | 1.64 |
| 2000 | 1.5 | 1.52 |
| 2500 | 1.2 | 1.23 |
| 3000 | 0.88 | 0.86 |
| 4000 | 0.22 | 0.25 |
| 5000 | -0.35 | -0.45 |

Table: 2 predicted and Measured values of bulk density at different depths

| Depth M | Predictive g/cm ³ | Measured values g/cm ³ |
|---------|------------------------------|-----------------------------------|
| 200 | 2.2 | 2.18 |
| 400 | 2.18 | 2.11 |
| 600 | 2.14 | 2.07 |
| 800 | 2.09 | 2.11 |
| 1200 | 1.96 | 1.95 |
| 1400 | 1.87 | 1.9 |
| 1600 | 1.77 | 1.75 |
| 1800 | 1.67 | 1.63 |
| 2000 | 1.57 | 1.6 |
| 2500 | 1.27 | 1.3 |
| 3000 | 0.95 | 0.92 |
| 4000 | 0.29 | 0.31 |
| 5000 | -0.29 | -0.24 |

Table: 3 predicted and Measured values of bulk density at different depths

| Depth M | Predictive g/cm ³ | Measured values g/cm ³ |
|---------|------------------------------|-----------------------------------|
| 200 | 2.32 | 2.29 |
| 400 | 2.33 | 2.35 |
| 600 | 2.33 | 2.37 |
| 800 | 2.34 | 2.31 |
| 1200 | 2.38 | 2.4 |
| 1400 | 2.39 | 2.43 |
| 1600 | 2.42 | 2.46 |
| 1800 | 2.45 | 2.48 |
| 2000 | 2.48 | 2.51 |
| 2500 | 2.57 | 2.63 |
| 3000 | 2.68 | 2.72 |
| 4000 | 2.96 | 2.97 |

| | | |
|-------------|-------------|-------------|
| 5000 | 3.31 | 3.36 |
|-------------|-------------|-------------|

Table: 4 predicted and Measured values of bulk density at different depths

| Depth M | Predictive g/cm³ | Measured values g/cm³ |
|----------------|------------------------------------|---|
| 200 | 2.02 | 1.99 |
| 400 | 1.99 | 1.91 |
| 600 | 1.95 | 1.88 |
| 800 | 1.91 | 1.95 |
| 1200 | 1.77 | 1.78 |
| 1400 | 1.69 | 1.72 |
| 1600 | 1.51 | 1.58 |
| 1800 | 1.49 | 1.52 |
| 2000 | 1.38 | 1.42 |
| 2500 | 1.09 | 1.07 |
| 3000 | 0.76 | 0.87 |
| 4000 | 0.1 | 0.2 |
| 5000 | -0.47 | -0.42 |

Table: 5 predicted and Measured values of bulk density at different depths

| Depth M | Predictive g/cm³ | Measured values g/cm³ |
|----------------|------------------------------------|---|
| 200 | 1.29 | 1.32 |
| 400 | 1.61 | 1.69 |
| 600 | 1.88 | 1.91 |
| 800 | 2.1 | 2.2 |
| 1200 | 2.42 | 2.49 |
| 1400 | 2.53 | 2.51 |
| 1600 | 2.6 | 2.55 |
| 1800 | 2.65 | 2.59 |
| 2000 | 2.67 | 2.71 |
| 2500 | 2.63 | 2.67 |
| 3000 | 2.51 | 2.65 |
| 4000 | 2.19 | 2.21 |
| 5000 | 2.17 | 2.21 |

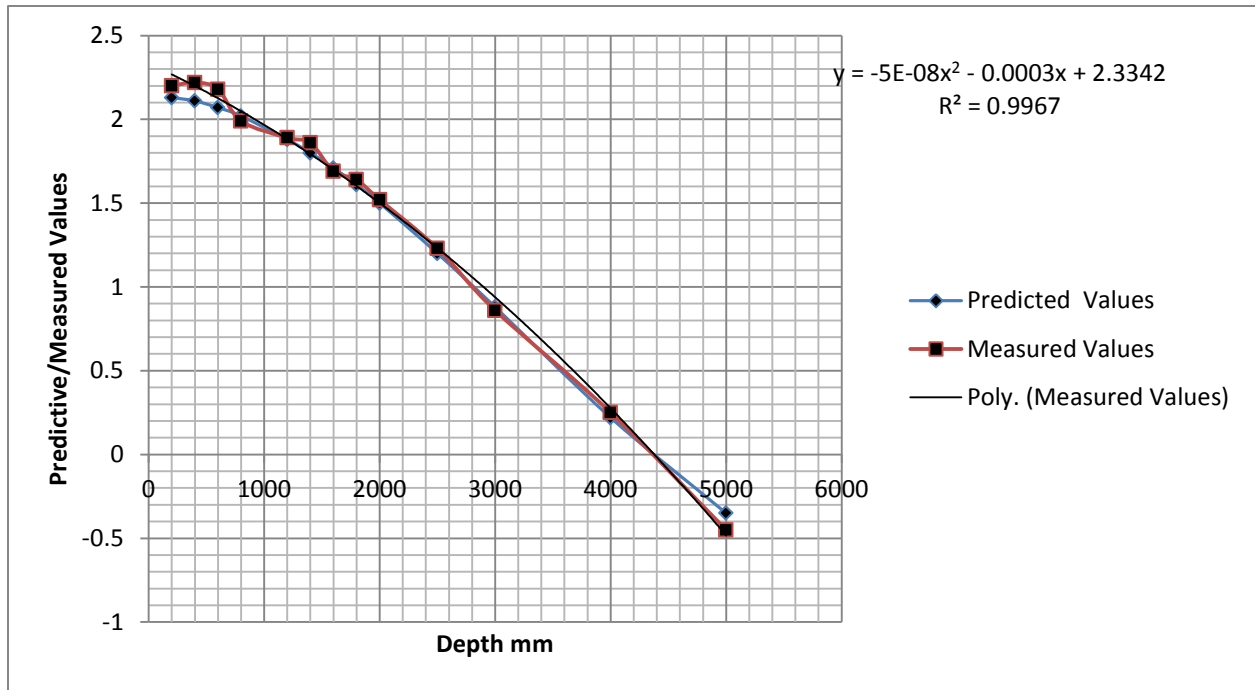


Figure: 1 predicted and Measured values of bulk density at different depths

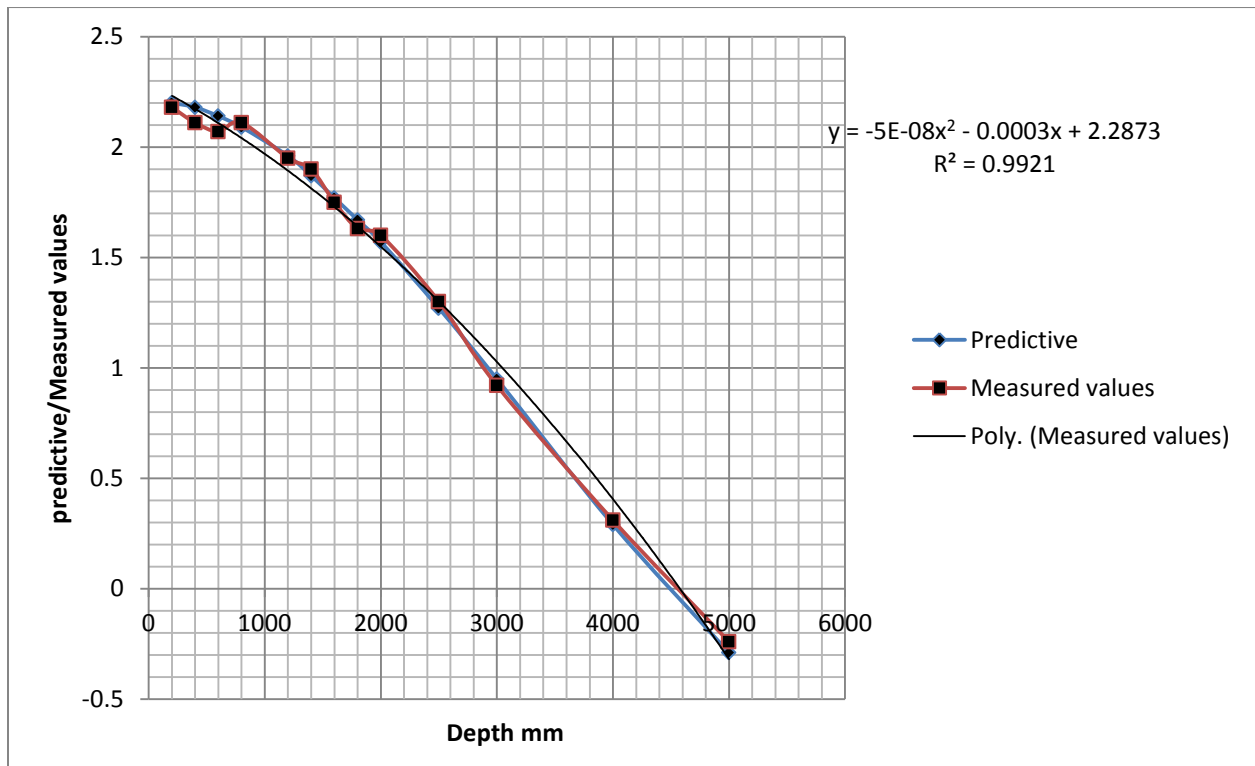


Figure: 2 predicted and Measured values of bulk density at different depths

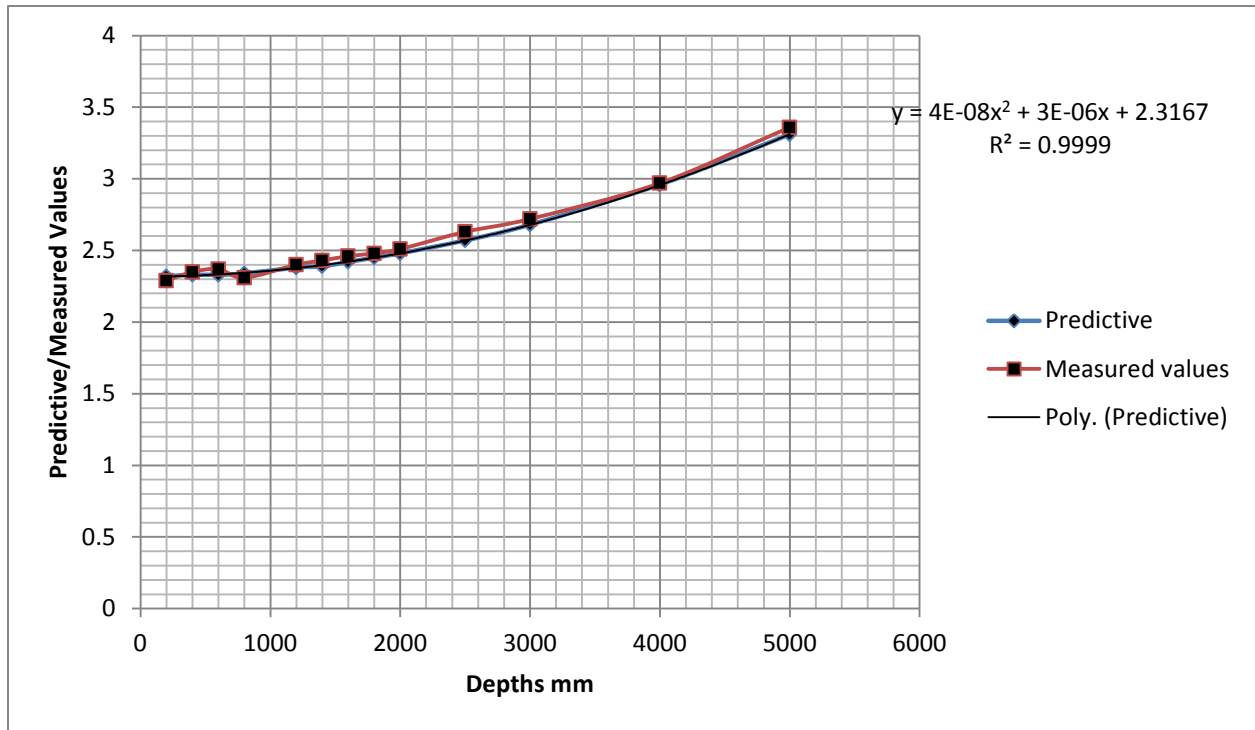


Figure: 3 predicted and Measured values of bulk density at different depths

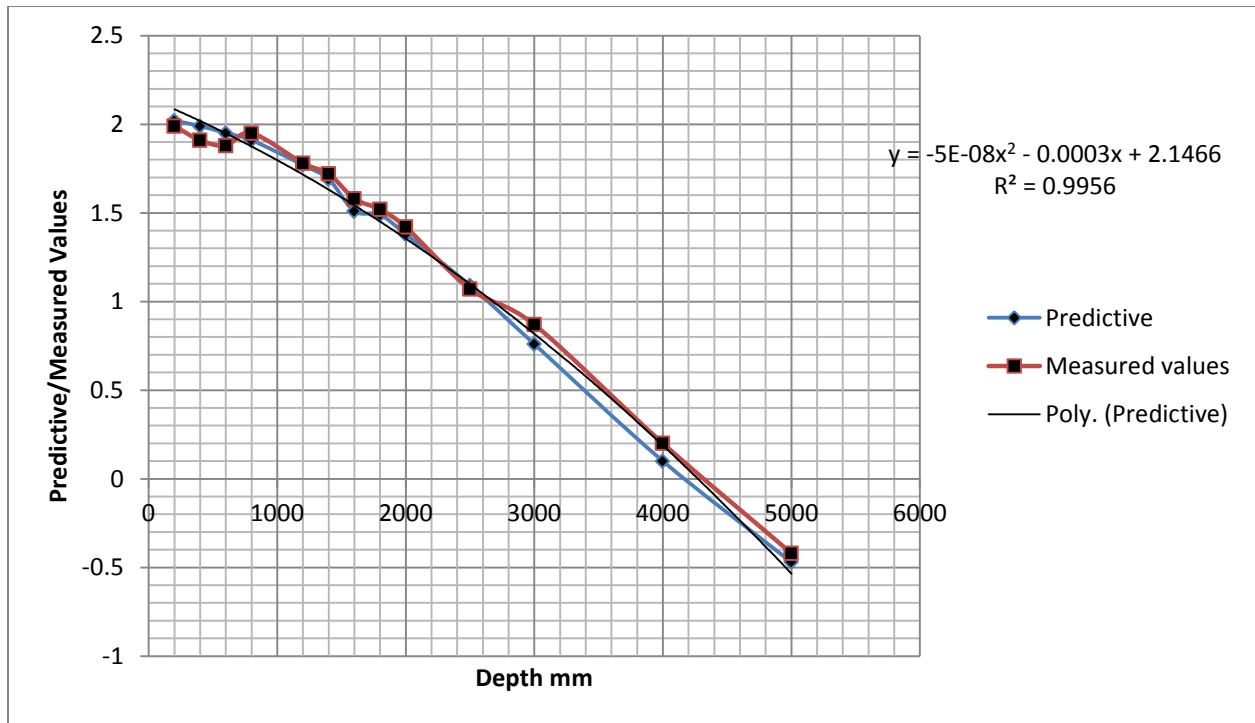


Figure: 4 predicted and Measured values of bulk density at different depths

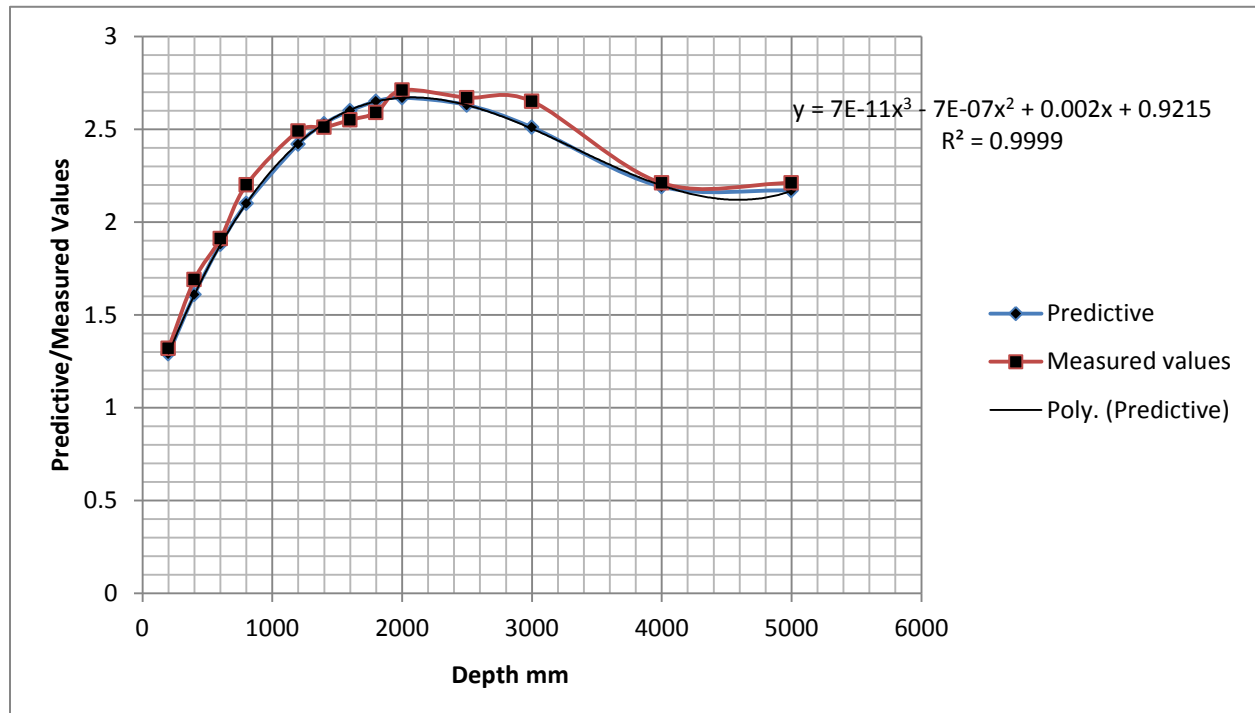


Figure: 5 predicted and Measured values of bulk density at different depths

The impact of the mechanism of the soil and their impact on the capability of soil to store water must be understood. The ability of the soil to accumulate water depends upon the volume of the voids present. Soil possessed of three major mechanisms: soil particles, air, and water. The fractions of water and air are contained in the voids between soil particles. The ratio of the volume of pores (voids) to the total (bulk) volume of a soil is the porosity (N). One way to determine porosity is to gauge the volume of a soil that is composed of soil particles and the fraction made up of the pores. The porosity may also be determined using the soil bulk density (Db). Bulk density is the density of the undisturbed (bulk) soil sample. Soil structure and texture largely determine bulk density. Soil structure refers to the arrangement of soil particles into secondary bodies called aggregates. Figure on shows high trend were observed at 200 mm, gradual decrease with increase in depths observed from 400mm to 5000mm, the expressed parameters shows the range being below average bulk densities, similar condition were observed on measured values it maintained the same trend as expressed from the figure developing a favourable fit. Figure two maintained the same trend similar to figure one that produced high bulk density at 200 mm, with gradual decrease down to the lowest at 5000mm, while that of the measured values expressed the same trend, both parameters produced the best fit, figure three express different condition from figure two below average densities were found between 200 and 2000mm the behaviour of the soil is through change in structural deposition of the formations, this condition are reflected on the parameter values expressed in the figure similarly, the measured values deposited the same formations displaying a

favorable fit, while figure four obtained its optimum values at 200mm just like figure one and two, the lowest rate of bulk density were recorded at 5000mm, while the measured values express similar trend, it obtained its optimum value at 200mm and gradually decreases with increase in depths to where the lowest were obtained. Figure five were similar to figure three the lowest were recorded at 200mm, gradual increase were observed to the optimum value at 200mm, fluctuation deposited between 3000mm and 5000mm, while that of the measured values maintained the same condition displaying the best favorable fits. The range are below the normal values, this implies that the Values lies between fine and coarse formation with slight location where the formations varies with slight loamy soil base on the coastal influences in the study location.

4. Conclusion

Soil particle density (g / cm^3) is accumulation of soil solids (oven-dry) per unit quantity of soil solids. Subdivision density depends on the densities of the various element solids and their relative large quantity. The elements density of most mineral soils ranges between 2.5 and 2.7 g / cm^3 . The series is fairly narrow because common soil minerals differ little in density. An average value of 2.65 g / cm^3 is often assumed. In dissimilarity, organic soils have lower particle densities since the density of organic matter is much less than that of mineral particles. In standard preparation, it will determine the particle density of a particular soil. It has been confirmed that it is easy to calibrate the mass of a small sample of soil but not so simple to precisely gauge the quantity of soil solids that make up this mass. Momentarily, the quantity of a known mass of soil solids is determined by indirectly measuring the quantity of water displaced by the soil solids. The mass of water displaced is actually measured, then the equivalent volume establish from the recognized density of water. Since fine-textured soils normally have more total pore space than coarse-textured soils, the finer soils also usually have lower bulk densities. Bulk density values of fine-textured soils commonly range from 1.0 to 1.3 g / cm^3 , while those of sandy soils range from about 1.3 to 1.7 g / cm^3 . Regardless of this universal disparity in bulk density between sandy and clayey soils, sandy soils are referred to as Light and clayey soils as Heavy. This terminology refers to relative ease of tillage, not typical bulk densities. Soil compaction, due to traffic from machinery or livestock or due to natural processes, decreases soil pore space and, therefore, increases bulk density. Since clay particles are plate-like, clay soils can be readily compressed and molded. Such compressibility, together with the low bulk density of clay soils, allows for substantial increases in bulk density when clay soils are compacted. In contrast, sand grains cannot be molded together. Thus, compaction of sandy soils with relatively small porosities does not lead to as great of increase in bulk density as occurs when clay soils are compacted. Although fine-textured soils generally have lower bulk densities than coarse-textured soils, the opposite can be true in compacted soils.

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